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GRADE 10 - WORKSHEET 1

1. **"The product of two consecutive positive integers is divisible by 2". Is this statement true or false? Justify your answer.**

   Ans: True.

   Let the consecutive integers be *n* and *n+1*. One of them is even, so their product is divisible by 2.

2. **"The product of three consecutive positive integers is divisible by 6". Is this statement true or false? Justify your answer.**

   Ans: True.

   Let the consecutive integers be *n, n+1, n+2*. One is divisible by 2, and one by 3, so their product is divisible by $2\times3=6$

3. **Prove that $\sqrt{p}+\sqrt{q}$ is irrational, where p, q are primes**.

   Ans: Assume $\sqrt{p}+\sqrt{q}$ is rational. Then $\sqrt{p}=r-\sqrt{q}$ (where *r* is rational). Squaring both sides, leads to a contradiction because *p* and *q* are primes, hence irrationality.

4. **Two alarm clocks ring their alarms at regular intervals of 50 sec and 48 sec. If they first beep together at 12 noon.at what time will beep again for the first time?**

   Ans: LCM of 50 and 48 = $2^4\times3\times5=2400$ seconds = 40 minutes.

   They beep together again at **12:40 pm**.

5. **Show the reciprocal of 3 + 2√2 is an irrational number**.

   Ans: Reciprocal: $\frac{1}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}=3-2\sqrt{2}$

   Since $\sqrt{2}$ is irrational, $3-2\sqrt{2}$ is irrational.

6. Show that $(\sqrt{3}+\sqrt{5})^2$ is an irrational number.

Ans: $\left(\sqrt{3}+\ \sqrt{5}\right)^2 = 3+5+2\sqrt{15} = 8+2\sqrt{15}$

Since $\sqrt{15}$ is irrational, $\left(\sqrt{3}+\ \sqrt{5}\right)^2$ is irrational.

7. **Prove that $\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)^2$ is rational**.

Ans: $$\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)^2 = 2+\frac{1}{2}+2\times\sqrt{2}\times\frac{1}{\sqrt{2}} = \frac{5}{2}+2 = \frac{9}{2}$$

$9/\ 2$ is rational.

8. If h is the HCF of 56 and 72, find x and y satisfying $h = 56x + 72y$.

Ans: HCF of 56 and 72 = 8.

$h = 56x + 72y$

$\mathbf{8 = 56} \times 4 + 72\ \times (-3)$

$8 = 224 - 216$

$\therefore x = 4, y = -3$

9. Show that only one of the numbers $n, n+2$ and $n+4$ is divisible by 3.

Ans: Among *n*, *n+2*, *n+4* one must be divisible by 3 because they are 3 consecutive numbers.

n can be in the form of 3q, 3q + 1, 3q + 2

When n = 3q

n + 2 = 3q + 2

n + 4 = 3q + 4

n is only divisible by 3

When n = 3q + 1

n + 2 = 3q = 3

n + 4 = 3q + 5

n + 2 is divisible by 3

When n = 3q + 2

n + 2 = 3q + 4

n + 4 = 3q + 2 + 4 = 3q + 6

n + 4 is divisible by 3.

Therefore, we can conclude that one and only one out of n, n + 2 and n + 4 is divisible by 3

10. Prove that $n^2 - n$ is divisible by 2 for any positive integer n.

$n^2 - n = n(n-1)$. Since n and $n-1$ are consecutive integers, one is even, so the product is divisible by 2.

11. Find the LCM and HCF of 336 and 54 and verify that

$$HCF \times LCM = product\ of\ two\ numbers.$$

Ans: $336 = 2^4 \times 3 \times 7$

$54 = 2 \times 3^3$

$HCF = 2 \times 3 = 6$

$LCM = 2^4 \times 3^3 \times 7 = 3024$

Verification: $6 \times 3024 = 18144 = 336 \times 54$

12. Prove that √5 is an irrational number.

Assume √5 is rational:

√5 = p/q

gcd (p,q)=1.

Squaring: $5q^2 = p^2$

p^2 divisible by 5 which is a contradiction to gcd p,q)=1.

Hence √5 irrational.