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MATHEMATICS GRADE 10 - WORKSHEET 3 – ANSWER KEY

1. Solve the system

$$4x + 3y = 32 \quad \text{and} \quad 2x - 7y = -18.$$

Multiply the second equation by 2 to match the $4x$ term:

$$4x - 14y = -36.$$

Subtract this from the first equation:

$$(4x + 3y) - (4x - 14y) = 32 - (-36) \Rightarrow 17y = 68 \Rightarrow y = 4.$$

Substitute $y = 4$ into $4x + 3y = 32$:

$$4x + 3(4) = 32 \Rightarrow 4x + 12 = 32 \Rightarrow 4x = 20 \Rightarrow x = 5.$$

Answer: $\boxed{x = 5,\ y = 4}$.

2. Add the two equations:

$$(2x + 5y) + (4x - 5y) = 6xy - 3xy \Rightarrow 6x = 3xy.$$

This gives $6x = 3xy \Rightarrow 3x(2 - y) = 0$. So either $x = 0$ or $y = 2$.

- If $x = 0$: substitute into $2x + 5y = 6xy$ gives $5y = 0 \Rightarrow y = 0$. So one solution is $(0, 0)$ with sum $x + y = 0$.
- If $y = 2$: substitute into $2x + 5(2) = 6x(2) \Rightarrow 2x + 10 = 12x \Rightarrow 10 = 10x \Rightarrow x = 1$. So another solution is $(1, 2)$ with sum $x + y = 3$.

Possible sums: $\boxed{0 \text{ or } 3}$.

3. Two-digit number problem

> The difference between the digits of a two-digit number is 3. If we reverse the digits and multiply it by 4, we obtain 7 times the original number.

Let the tens digit be t and units digit u. Original number $= 10t + u$. Reversed $= 10u + t$.
Given $4(10u + t) = 7(10t + u) \Rightarrow 40u + 4t = 70t + 7u \Rightarrow 33u = 66t \Rightarrow u = 2t$.

Also the digits differ by 3: $|t - u| = 3$. Using $u = 2t$, we get either $2t - t = 3 \Rightarrow t = 3$ (so $u = 6$) or the other sign gives impossible negative digit. Thus digits are $t = 3, u = 6$. Number = **36**.

Check: Reverse $= 63,\ 4 \times 63 = 252,\ 7 \times 36 = 252$. ✓

Answer: $\boxed{36}$.

4. Price of chicken and duck

Equations from given sales:

$$50C + 30D = 550 \quad (1),$$

$$44C + 36D = 532 \quad (2).$$

Eliminate D. Multiply (1) by 6 and (2) by 5:

$$300C + 180D = 3300,$$

$$220C + 180D = 2660.$$

Subtract: $80C = 640 \Rightarrow C = 8$. Then $50 \cdot 8 + 30D = 550 \Rightarrow 400 + 30D = 550 \Rightarrow 30D = 150 \Rightarrow D = 5$.

Answer: $\boxed{\text{Chicken } = ₹8,\ \text{Duck } = ₹5}$.

5. Find $x + y$ for

$$10x + 3y = 75, \qquad 6x - 5y - 11 = 0\ (\Rightarrow 6x - 5y = 11).$$

Multiply first equation by 5: $50x + 15y = 375$. Multiply second by 3: $18x - 15y = 33$. Add:

$$68x = 408 \Rightarrow x = 6.$$

Then $10(6) + 3y = 75 \Rightarrow 60 + 3y = 75 \Rightarrow 3y = 15 \Rightarrow y = 5$.

Answer: $\boxed{x = 6,\ y = 5,\ \text{so } x + y = 11.}$

6. Step 1: Define Variables

Let the speed of X be s km/h and the speed of Y be t km/h.

Step 2: Write the Time Taken for Each Person

The time taken by X to walk 30 km is:

$$\textbf{Time taken by X} = \frac{30}{s}$$

The time taken by Y to walk 30 km is:

$$\textbf{Time taken by Y} = \frac{30}{t}$$

Step 3: Set Up the First Equation

According to the problem, X takes 3 hours more than Y to walk 30 km. Therefore, we can write the equation:

$$\frac{30}{s} - \frac{30}{t} = 3$$

Step 4: Simplify the First Equation

We can simplify this equation:

$$\frac{30}{s} - \frac{30}{t} = 3$$

Multiplying through by st (the product of both speeds) gives:

$$30t - 30s = 3st$$

Rearranging gives us:

$$3st = 30t - 30s$$

Dividing everything by 3:

$$st = 10t - 10s$$

This can be rearranged to form our first equation:

$$\frac{1}{s} - \frac{1}{t} = \frac{1}{10} \quad \textbf{(Equation 1)}$$

Step 5: Set Up the Second Equation

Now, if X doubles his speed, his new speed becomes $2s$. The time taken by X at this new speed is:

$$\text{Time taken by X at double speed} = \frac{30}{2s} = \frac{15}{s}$$

According to the problem, when X doubles his speed, he is ahead of Y by 1.5 hours. Thus, we can write the second equation:

$$\frac{30}{t} - \frac{15}{s} = 1.5$$

Step 6: Simplify the Second Equation

We can simplify this equation:

$$\frac{30}{t} - \frac{15}{s} = 1.5$$

Multiplying through by $2st$ gives:

$$60s - 30t = 3st$$

Rearranging gives us:

$$3st = 60s - 30t$$

Dividing everything by 3:

$$st = 20s - 10t$$

This can be rearranged to form our second equation:

$$\frac{1}{t} - \frac{1}{2s} = \frac{1}{20} \quad \textbf{(Equation 2)}$$

Step 7: Solve the System of Equations

Now we have two equations:

1. $\frac{1}{s} - \frac{1}{t} = \frac{1}{10}$

2. $\frac{1}{t} - \frac{1}{2s} = \frac{1}{20}$

We can add these two equations to eliminate $\frac{1}{t}$:

$$\left(\frac{1}{s} - \frac{1}{t}\right) + \left(\frac{1}{t} - \frac{1}{2s}\right) = \frac{1}{10} + \frac{1}{20}$$

This simplifies to:

$$\frac{1}{s} - \frac{1}{2s} = \frac{1}{10} + \frac{1}{20}$$

Combining the left side gives:

$$\frac{1}{2s} = \frac{3}{20}$$

Cross-multiplying gives:

$$20 = 6s \implies s = \frac{20}{6} = \frac{10}{3} \text{ km/h}$$

Step 8: Find the Speed of Y

Now, substitute $s = \frac{10}{3}$ back into Equation 1:

$$\frac{1}{\frac{10}{3}} - \frac{1}{t} = \frac{1}{10}$$

This simplifies to:

$$\frac{3}{10} - \frac{1}{t} = \frac{1}{10}$$

Rearranging gives:

$$\frac{1}{t} = \frac{3}{10} - \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$$

Thus, $t = 5$ km/h.

Conclusion

The speeds of X and Y are:

- Speed of X = $\frac{10}{3}$ km/h

- Speed of Y = 5 km/h

7. Mixing vessels to get 25 L milk & 10 L water (third vessel)

Vessel I: 24 L milk, 6 L water → total 30 L, milk fraction $= \frac{24}{30} = 0.8$.

Vessel II: 15 L milk, 10 L water → total 25 L, milk fraction $= \frac{15}{25} = 0.6$.

Let x litres taken from Vessel I and y litres from Vessel II. We need total $x + y = 35$ litres with milk amount 25 L:

$$0.8x + 0.6y = 25, \qquad y = 35 - x.$$

Substitute:

$$0.8x + 0.6(35 - x) = 25 \Rightarrow 0.8x + 21 - 0.6x = 25 \Rightarrow 0.2x = 4 \Rightarrow x = 20.$$

So $y = 35 - 20 = 15$.

Answer: $\boxed{20 \text{ L from vessel I and } 15 \text{ L from vessel II.}}$

8. Graphs of lines and finding a

Lines:

$$(1) \ -x + 3y = 6, \qquad (2) \ 2x - 3y = 12.$$

Add (1) and (2):

$$(-x + 3y) + (2x - 3y) = 6 + 12 \Rightarrow x = 18.$$

Substitute $x = 18$ into (1):

$$-18 + 3y = 6 \Rightarrow 3y = 24 \Rightarrow y = 8.$$

Intersection point is $(18, 8)$.

Now find a from $3x + 2y = 3 + a$ by substituting $(18, 8)$:

$$3(18) + 2(8) = 54 + 16 = 70 = 3 + a \Rightarrow a = 67.$$

Area of triangle formed by these two lines and the Y-axis

The two lines meet the Y-axis at $x = 0$:

- For (1): $-0 + 3y = 6 \Rightarrow y = 2$ → point $(0, 2)$.
- For (2): $2 \cdot 0 - 3y = 12 \Rightarrow y = -4$ → point $(0, -4)$.

Triangle vertices: $(0, 2), \ (0, -4), \ (18, 8)$. Base on Y-axis length $= 2 - (-4) = 6$. Height (horizontal distance from Y-axis to vertex) $= 18$.

Area $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 6 \times 18 = 54$.

Answers: $\boxed{a = 67, \ \text{Area} = 54 \text{ sq. units}}$.

9. Acid mixture: make 10 L of 40% from 50% and 25%

Let x L of 50% and y L of 25%; $x + y = 10$. Acid equation:

$$0.50x + 0.25y = 0.40 \times 10 = 4.$$

Substitute $y = 10 - x$:

$$0.5x + 0.25(10 - x) = 4 \Rightarrow 0.5x + 2.5 - 0.25x = 4 \Rightarrow 0.25x = 1.5 \Rightarrow x = 6.$$

So $y = 4$.

10. Boat upstream/downstream speeds

Let speed of boat in still water $= u$ km/h and stream speed $= v$ km/h. Then upstream speed $= u - v$, downstream $= u + v$.

Given

$$\frac{32}{u - v} + \frac{36}{u + v} = 7 \quad (1),$$

$$\frac{40}{u - v} + \frac{48}{u + v} = 9 \quad (2).$$

Let $p = \frac{1}{u - v}$, $q = \frac{1}{u + v}$. Then

$$32p + 36q = 7, \qquad 40p + 48q = 9.$$

Multiply first by 5: $160p + 180q = 35$. Multiply second by 4: $160p + 192q = 36$. Subtract:

$$12q = 1 \Rightarrow q = \frac{1}{12} \Rightarrow u + v = 12.$$

Substitute q into $32p + 36q = 7$:

$$32p + 36 \cdot \frac{1}{12} = 7 \Rightarrow 32p + 3 = 7 \Rightarrow 32p = 4 \Rightarrow p = \frac{1}{8} \Rightarrow u - v = 8.$$

Now solve

$$u + v = 12, \qquad u - v = 8.$$

Add: $2u = 20 \Rightarrow u = 10$. Then $v = 12 - 10 = 2$.

Answer: $\boxed{\text{Boat speed in still water } = 10 \text{ km/h}, \text{ stream speed} = 2 \text{ km/h}.}$